Performance Evaluation of New Energy Detection Based Spectrum Sensing Methods in Cognitive Radio

[1]Sumit Lohan, [2]Rita Mahajan
[1,2]E & EC Dept., PEC University of Technology
[1]lohansumit86@gmail.com, [2]ritamahajan@pec.ac.in

Abstract--For Cognitive Radio, energy detection is the most popular spectrum sensing method since other spectrum sensing methods like matched filter detection, cyclostationary feature detection etc. need to know the prior information i.e. frequency, phase, modulation scheme etc. about the primary signal whereas energy detection does not and it is also simpler to implement. In this paper, we compare the ROC (Receiver Operating Characteristics) curves and probability of detection (P_d) versus SNR (Signal-to-Noise Ratio) curves for AWGN channel using squaring, cubing and double-squaring operations. The cubic operation shows an improvement of up to 1.3 times and double-squaring operation shows an improvement of up to 2 times as compared to the squaring operation for AWGN channel.

Keywords--Cognitive Radio, Spectrum Sensing, SNR, AWGN channel.

1 INTRODUCTION

In today's world, due to increasing interest of people in wireless communication and its progressive growth, the demand of electromagnetic spectrum is increasing day-by-day which has led to under-utilization of the spectrum. A fixed portion of the total available EM spectrum is assigned to a user which leads to wastage of a large portion of it because the users other than the licensed user cannot use it. To mitigate this problem of spectrum scarcity, Federal Communications Commission (FCC) has proposed the solution by allowing the unlicensed users to use the licensed bands opportunistically and named it as Cognitive Radio (CR) [1]. Cognitive Radio is an intelligent wireless technology due to its ability to rapidly and autonomously adapt operating parameters to changing environment [2]. The two main characteristics of Cognitive Radio are: Cognitive Capability (i.e. gathering the information about the environment) and reconfigurability (i.e. adapting its transmission parameters according to the gathered information) [3]. One of the most important tasks that cognitive radio performs is spectrum sensing. In cognitive radio, spectrum sensing aims at monitoring the usage and the characteristics of the covered spectral band(s) and is thus required by the secondary users both before and during the use of licensed spectral bands [1]. Spectrum sensing is not an easy task because of shadowing, fading and time-varying natures of wireless channels. There are many spectrum sensing techniques available like energy detection, matched filter coherent detection, cyclostationary feature detection etc. [4].

Among all the above specified spectrum sensing techniques, energy detection is the most popular technique as it is of non-coherent type and has low implementation complexity. This energy detection technique, also called radiometry or periodogram does not require any prior knowledge of primary user's signal [5]. In this method, we measure the energy of the received signal and compare it with a predefined threshold to determine the presence or absence of primary user's signal. Moreover, energy detector is mainly used in ultra wideband communication to borrow an idle channel from licensed user. In this paper, probability of detection (P_d), probability of false alarm (P_f) and probability of missed detection ($P_m = 1 - P_d$) are the key measurement metrics that analyze the performance of an energy detector. The performance of an energy detector is illustrated by probability of detection (P_d) versus SNR curves and the receiving operating characteristics (ROC) curves which is a plot of P_d versus P_f or P_m versus P_f [6].

This paper is organized as follows: Section 2 describes the system model and a list of important notations. In section 3, probability of detection and false alarm of non-fading AWGN channel is described. Simulation result for squaring, cubing and double-squaring operations and their comparison are illustrated in section 4 followed by conclusions in section 5.

2 SYSTEM MODEL AND NOTATIONS

First of all, before describing the system model, here we list the main notations which are used in this paper for additional clarity and to avoid any kind of confusion.

- $s(t)$: primary user's transmitted signal
- $y(t)$: received signal
- $n(t)$: additive white Gaussian noise
- h : amplitude gain of the channel
- N_{01} : one-sided noise power spectral density
- E_s : signal energy= $\int_0^T s^2(t)\, dt$
- $\gamma = \frac{E_s}{N_{01}}$: signal-to-noise ratio (SNR)
- λ : energy threshold used by the energy detector
- T : observation time interval in second
- W : one-sided bandwidth (Hz) i.e. positive bandwidth of the low-pass signal
- $u = TW$: time-bandwidth product
- f_c : carrier frequency
- P_d : probability of detection
- P_f : probability of false alarm
- $P_m = 1 - P_d$: probability of missed-detection
- H_0 : hypothesis 0 corresponding to no signal transmitted
- H_1 : hypothesis 1 corresponding to signal transmitted
- $N(\mu, \sigma^2)$: a Gaussian variate with mean μ and variance σ^2
- χ^2_α : a central chi-square variate with α degree of freedom
- $\chi^2_\alpha(\beta)$: a non-central chi-square variate with α degree of freedom and non-centrality parameter β

To detect the energy of the received signal, an energy detector is used by each CR user [7]. Energy detector consists of four main blocks [8]:

1. Noise pre-filter
2. A/D converter (Analog-to-Digital Converter)
3. Squaring Device
4. Integrator

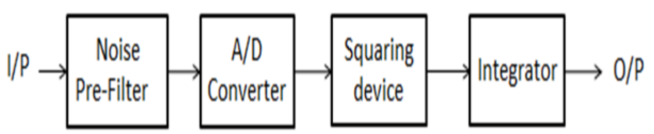


Fig. 1: Block diagram of Energy detector

The output of the integrator at any time is the energy of the filter received signal over the time interval T. The noise pre-filter limits the noise bandwidth; the noise at the input to the squaring device has a band-limited, flat spectral density. The output of the integrator is considered as the test statistic to test the two hypotheses H_0 and H_1 [7].

The received signal $y(t)$ takes the form

$$y(t) = h\, s(t) + n(t) \tag{1}$$

where $h = 0$ or 1 under hypotheses H_0 or H_1, respectively. As described in [7], the received signal first pre-filtered by an ideal band-pass filter with transfer function [9][10]

$$H(f) = \begin{cases} \frac{2}{\sqrt{N_{01}}}, & |f - f_c| \le W \\ 0 \quad , & |f - f_c| > W \end{cases} \tag{2}$$

to limit the average noise power and normalize the noise variance. The output of this filter is then squared and integrated over a time interval T to finally produce a measure of the energy of the received waveform. The output of the integrator denoted by Y as in fig. 1 will act as the test statistic to test the two hypotheses H_0 and H_1. According to the sampling theorem, the noise process [11] can be expressed as

$$n(t) = \sum_{i=-\infty}^{\infty} n_i \operatorname{sin} c(2Wt - i) \tag{3}$$

where $\operatorname{sin} c(x) = \frac{\sin(\pi x)}{\pi x}$ and $n_i = n\left(\frac{i}{2W}\right)$.

One can easily check that $n_i \sim N(0, N_{01} W)$, for all i.

Over the time interval $(0, T)$, the noise energy can be approximated as [7].

$$\int_0^T n^2(t)\, dt = \frac{1}{2W} \sum_{i=1}^{2u} n_i^2 \tag{4}$$

where $u = TW$. We assume that T and W are chosen to restrict u to integer values. If we define

$$n'_i = \frac{n_i}{\sqrt{N_{01}W}}, \tag{5}$$

then, the test or decision statistic Y can be written as [7]

$$Y = \sum_{i=1}^{2u} n_i'^2 \tag{6}$$

Y can be viewed as the sum of the squares of $2u$ standard Gaussian variates with zero mean and unit variance. Therefore, Y follows [15] a central chi-square (χ^2) distribution with $2u$ degrees of freedom. The same approach is applied when the signal $s(t)$ is present with the replacement of each n_i by $n_i + s_i$ where $s_i = s\left(\frac{i}{2W}\right)$. The decision statistic Y in this case will have a non-central χ^2 distribution with $2u$ degrees of freedom and a non-centrality parameter 2γ [7]. Following the shorthand notations mentioned in the beginning of this section, we can describe the decision statistic as

$$Y \sim \begin{cases} \chi^2_{2u}, & H_0 \\ \chi^2_{2u}(2\gamma), & H_1 \end{cases} \tag{7}$$

The probability density function (PDF) [9] of Y can be written as

$$f_Y(y) = \begin{cases} \frac{1}{2^u \Gamma(u)} y^{u-1} e^{-\frac{y}{2}}, & H_0 \\ \frac{1}{2}\left(\frac{y}{2\gamma}\right)^{\frac{u-1}{2}} e^{-\frac{2\gamma+y}{2}} I_{u-1}\left(\sqrt{2\gamma y}\right), & H_1 \end{cases} \tag{8}$$

where $\Gamma(.)$ is the gamma function [12, section 8.31] and $I_v(.)$ is the v-th-order modified Bessel function of the first kind [12, section 8.43].

3 PROBABILITY OF DETECTION AND FALSE ALARM OF NON-FADING AWGN CHANNEL

An approximate expression for probability of detection for non-fading AWGN channel was presented in [7]. In this section, we present exact closed-form expression for both probability of detection P_d and probability of false alarm P_f. The P_d and P_f can be generally evaluated by

$$P_d = \Pr(Y > \lambda \mid H_1), \tag{9}$$

$$P_f = \Pr(Y > \lambda \mid H_0), \tag{10}$$

where λ is decision threshold. Also, P_f can be written in terms of Probability density function (PDF) as: [13, Eq. (4-16) and Eq. (4-22)]

$$P_f = \int_{\lambda}^{\infty} f_Y(y)\, dy \tag{11}$$

using equation(11),

$$P_f = \frac{1}{2^u \Gamma(u)} \int_{\lambda}^{\infty} y^{u-1} e^{-\frac{y}{2}} dy \tag{12}$$

Dividing and multiplying the R.H.S. of above equation by 2^{u-1}, we get

$$P_f = \frac{1}{2\Gamma(u)} \int_{\lambda}^{\infty} \left(\frac{y}{2}\right)^{u-1} e^{-\frac{y}{2}} dy \tag{13}$$

substituting $\frac{y}{2} = t$, $\frac{dy}{2} = dt$ and changing the limit of integration to $\left(\frac{\lambda}{2}, \infty\right)$, we get

$$P_f = \frac{1}{\Gamma(u)} \int_{\lambda/2}^{\infty} (t)^{u-1} e^{-(t)} dt \tag{14}$$

or,

$$P_f = \frac{\Gamma(u, \lambda/2)}{\Gamma(u)} \tag{15}$$

Now probability of detection can be written by making use of the cumulative distribution function [13, Eq. (4-22)].

$$P_d = 1 - F_Y(y) \tag{16}$$

The cumulative distribution function (CDF) of Y can be obtained (for an even number of degree of freedom which is $2u$ in our case) as [14, Eq. (2.1-124)]:

$$F_Y(y) = 1 - Q_u(\sqrt{\lambda}, \sqrt{y}) \tag{17}$$

where $Q_u(a,b)$ is the generalized Marcum Q-function [15]. Hence,

$$P_d = Q_u(\sqrt{2\gamma}, \sqrt{\lambda}) \tag{18}$$

4 SIMULATION RESULT

The performance of energy detector is analyzed using probability of detection P_d versus SNR curves and ROC (Receiving Operating Characteristic) curves for an AWGN channel. Monte-Carlo method is used for simulation. Fig. 2 and Fig. 3 depict the P_d versus SNR curves for different values of probability of false alarm (P_f) and ROC curves for different values of SNR for the squaring operation of an AWGN channel, respectively. It can be seen in these figures that with increase in SNR (Signal to Noise Ratio), the performance of energy detector improve. Fig. 4 depict the comparison of P_d versus SNR curves for squaring, cubing and double-squaring operations and the comparison of ROC curves for all the three operations is shown in Fig. 5

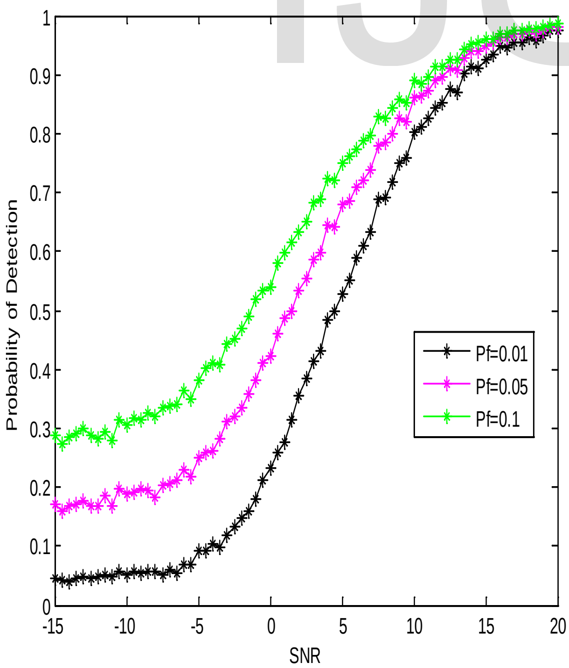


Fig. 2: P_d versus SNR curve for AWGN channel using squaring operation

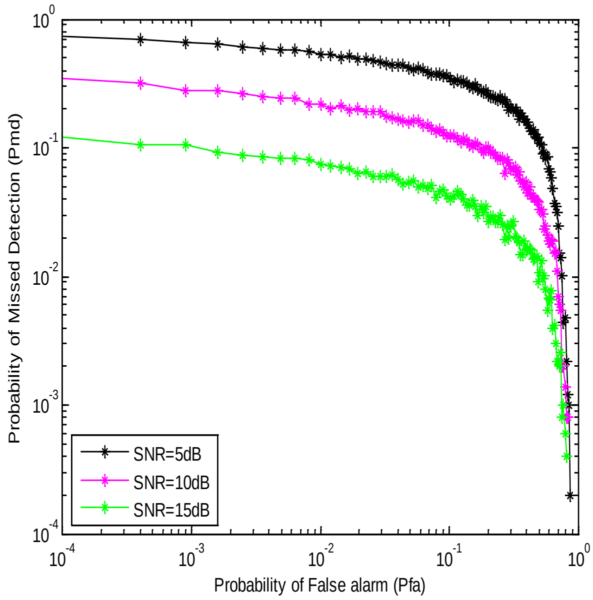
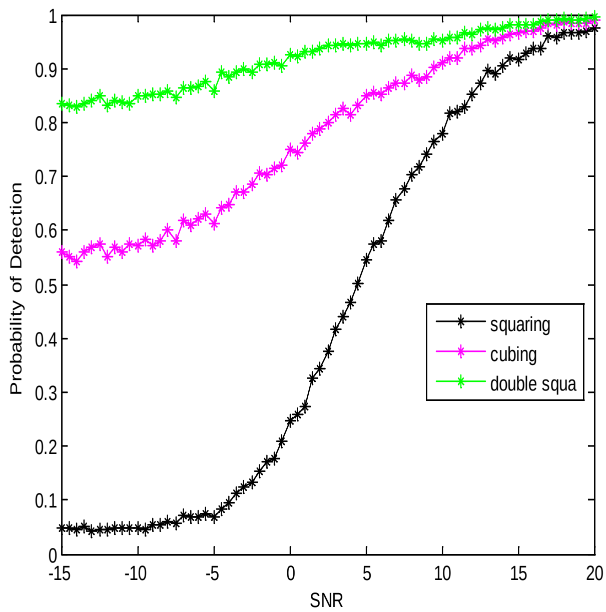


Fig. 3: Complementary ROC curve for AWGN channel using squaring operation

Fig. 4 and Fig. 5 depict improvement in the performance of the energy detector using cubic and double-squaring operations over AWGN channel. The results obtained are quantified as shown in TABLE 1 and TABLE 2. These results illustrate improvement in the probability of detection using cubic and double-squaring operations. This improvement has gone up to 1.3 times for cubic operation and 2 times for double-squaring operation as compared to squaring operation for AWGN channel. We assume time-bandwidth product=5 and average SNR=5 db.



ISSN 2229-5518

Fig. 4: Comparison of P_d versus SNR curves for AWGN channel

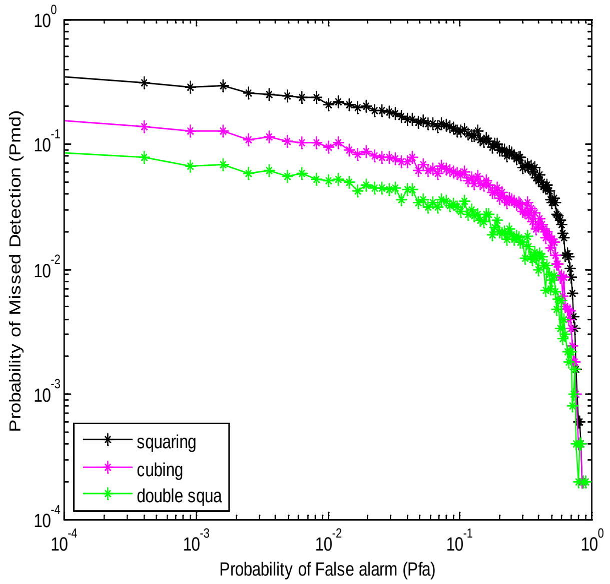


Fig. 5: Comparison of complementary ROC curves for AWGN channel

5 CONCLUSIONS

We have discussed the energy detection spectrum sensing technique in cognitive radio networks. Energy detection has the advantage of low implementation and computational complexities. In the present work, the performance of energy detector is analyzed. Closed form expression for probability of detection and false alarm over AWGN channel are described. It is shown by using ROC curves that probability of detection improves if we use cubing or double-squaring operation instead of squaring operation.

Probability of false alarm	**Probability of detection for AWGN Channel (Squaring Device)**	**Probability of detection for AWGN Channel (Cubing Device)**	**Improvement (times)**
0.0001	0.2844	0.6568	1.3094
0.0196	0.5570	0.8358	0.5005
0.1600	0.7680	0.9252	0.2046
0.4096	0.9150	0.9766	0.0673
0.9025	0.9988	0.9998	0.0010

TABLE 1: Improvement using cubing operation for AWGN channel.

Probability of false alarm	**Probability of detection for AWGN Channel (Squaring Device)**	**Probability of detection for AWGN Channel (Double-squaring Device)**	**Improvement (times)**
0.0001	0.2844	0.8506	1.9908
0.0196	0.5570	0.9368	0.6818
0.1600	0.7680	0.9734	0.2674
0.4096	0.9150	0.9896	0.0815
0.9025	0.9988	1.0000	0.0012

TABLE 2: Improvement using double-squaring operation for AWGN channel.

REFERENCES

[1] J. Ma, G. Y. Li, B.H. Juang, "Signal Processing in Cognitive Radio." Proceedings of the IEEE, Vol. 97, pp. 805-823, May 2009.

[2] H. A. Mahmoud, T. Yucek and H. Arslan. "OFDM For Cognitive Radio-Merits and Challenges." IEEE Wireless Communications, Vol. 16, pp. 6-15, April 2009.

[3] I. F. Akyildiz, W.Y.Lee, M.C. Vuran, S. Mohanty. "Next Generation/Dynamic Spectrum Access/Cognitive Radio Wireless Networks: A Survey." Comp. Net. J., Vol. 50, pp. 2127–59, Sept. 2006.

[4] I. F. Akyildiz, W.-Y. Lee, M. C. Vuran, and S. Mohanty, "Next generation/dynamic spectrum access/cognitive radio wireless networks: a survey," Computer Networks, Vol. 50, pp. 2127–2159, May 2006.

[5] T. Yucek and H. Arslan. "A Survey of Spectrum Sensing Algorithms for Cognitive Radio Applications." IEEE Communications Surveys & Tutorials, Vol. 11, pp.116-130, Jan. 2009.

[6] S. Atapattu, C. Tellambura, and H. Jiang. "Energy detection of primary signals over η-μ fading channels." in Proc. Fourth International Conference on Industrial and Information Systems, ICIIS, pp. 118-122, 2009.

[7] H. Urkowitz, "Energy detection of unknown deterministic signals," Proc. IEEE, Vol. 55, pp. 523–531, April. 1967.

[8] S. Ciftci and M. Torlak. "A Comparison of Energy Detectability Models for Spectrum Sensing." in IEEE GLOBECOM, pp.1-5, 2008.

[9] F. F. Digham, M. -S. Alouini, and M. K. Simon, "On the energy detection of unknown signals over fading channels," in Proc. IEEE Int. Conf. on Communication, Vol. 5, pp. 3575–3579, May 2003.

[10] V. I. Kostylev, "Energy detection of a signal with random amplitude," in Proc. IEEE Int. Conf. on Communication. (ICC'02), New York City, New York, pp. 1606–1610, May 2002.

[11] C.E. Shannon, "Communication in the presence of noise," Proc. IRE, Vol. 37, pp. 1-21, January 1949.

[12] I. S. Gradshteyn and I. M. Ryzhik, Table of Integrals, Series, and Products. San Diego, CA: Academic Press, sixth ed., 2000.

[13] A. Papoulis and S.U Pillai, Probability, Random Variables, and Stochastic Processes, New York: McGraw-Hill, ed. 4th, 2002.

[14] J. G. Proakis, Digital Communications, New York: McGraw-Hill, ed. 4th, 2001.

[15] A. H. Nuttall, "Some integrals involving the QM-function," Naval Underwater Systems Center (NUSC) technical report, May 1974.